ECE 373 Fundamentals of Communication

Lecture-17

Phase Modulation

For the modulating signal, i.e., message signal, $m(t)$, the phase modulated (PM) is obtained as

$$s(t) = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$$

Frequency Modulation

For the modulating signal, i.e., message signal, $m(t)$, the frequency modulated (FM) is obtained as

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right)$$

Narrowband and Wideband FM

For message signal $m(t)$, narrowband FM is generated as

$$s(t) = A_c \cos(2\pi f_c t) - A_c\theta(t)\sin(2\pi f_c t)$$

$$\theta(t) = 2\pi k_f \int_0^t m(t)dt$$

For narrowband modulated signals we have

$$\beta = \frac{\Delta f}{W_m} \ll 1$$

and for wideband modulated signals we have

$$\beta = \frac{\Delta f}{W_m} > 1.$$

For the message signal $m(t)$ having bandwidth W_m, the bandwidth of the narrowband modulated signal $s_{NBFM}(t)$ equals to $2W_m$.

Wideband FM (WBFM) Bandwidth

The FM signal is given as

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right).$$

If the bandwidth if the message signal $m(t)$ is W_m, then the bandwidth of the integrate signal

$$\int_0^t m(t)dt$$

is also W_m. Note that

$$\int_{-\infty}^t m(t)dt \overset{FT}{\leftrightarrow} \frac{1}{j2\pi f} M(f) + \frac{1}{2}\delta(f)M(0).$$

Carson's Rule

The bandwidth if the WBFM signal $s(t)$ can be calculated as

$$BW = 2(\Delta f + W_m)$$

where Δf is the frequency deviation and W_m is the bandwidth of the message signal $m(t)$.

Note that for FM signal the frequency deviation Δf is calculated as

$$\Delta f = k_f \max|m(t)|.$$

Generation of FM Signals

FM signals can be generated using Direct and Indirect Generation Methods.

Direct Generation of FM Signals

One method for generating an FM signal directly is to design an oscillator whose frequency changes with the input voltage.

When the input voltage is zero, the oscillator generates a sinusoid with frequency fc, and when the input voltage changes, this frequency changes accordingly.

There are two approaches to designing such an oscillator, usually called a VCO or voltage-controlled oscillator. One approach is to use a varactor diode.

A varactor diode is a capacitor whose capacitance changes with the applied voltage.

Therefore, if this capacitor is used in the tuned circuit of the oscillator and the message signal is applied to it, the frequency of the tuned circuit, and the oscillator, will change in accordance with the message signal.

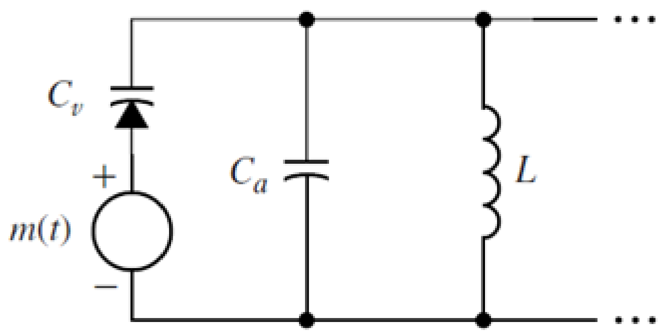


Figure-1 FM Signal Generation Using Varactor Diode

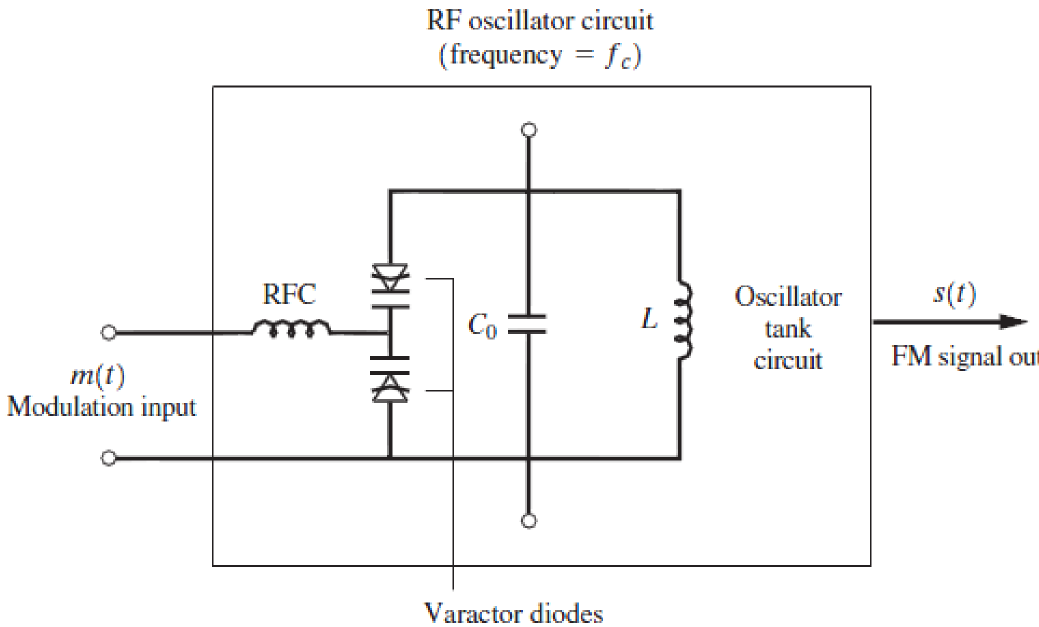
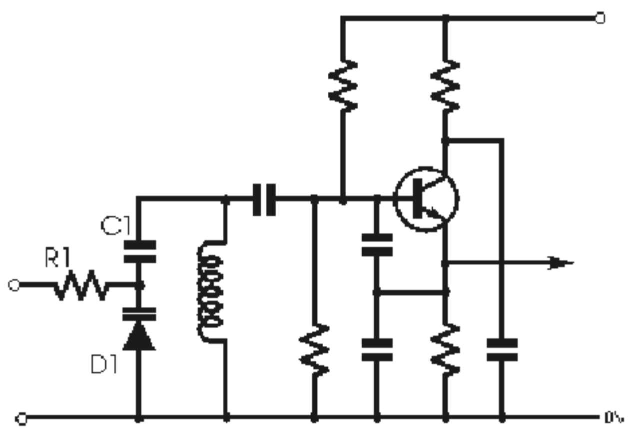


Figure-1A

The inductor is part of an LRC tank circuit whose frequency is determined by the capacitance on D1, which depends on the input voltage.



This is an example of a Hartley oscillator. There are lots of other circuits that have been invented for FM modulation.

Figure-1B

For Figure-1, the capacitance of the varactor diode is given by

$$C(t) = C_0 + k_0 m(t).$$

The value of the inductor is L_0.

When $m(t) = 0$, the frequency of the tuned circuit is given by $f_c = \frac{1}{2\pi\sqrt{L_0 C_0}}$. In general, for nonzero $m(t)$, we have

$$\begin{aligned} f_i(t) &= \frac{1}{2\pi\sqrt{L_0(C_0 + k_0 m(t))}} \\ &= \frac{1}{2\pi\sqrt{L_0 C_0}} \frac{1}{\sqrt{1 + \frac{k_0}{C_0} m(t)}} \\ &= f_c \frac{1}{\sqrt{1 + \frac{k_0}{C_0} m(t)}} \end{aligned}$$

Assuming that

$$\epsilon = \frac{k_0}{C_0} m(t) \ll 1$$

and using the approximations

$$\begin{aligned} \sqrt{1+\epsilon} &\approx 1 + \frac{\epsilon}{2} \\ \frac{1}{1+\epsilon} &\approx 1 - \epsilon, \end{aligned}$$

we obtain

$$f_i(t) \approx f_c \left(1 - \frac{k_0}{2C_0} m(t)\right)$$

which is the relation for a frequency-modulated signal.

A serious limitation of the direct method is the tendency for the carrier frequency to drift.

This method of FM generation is more suitable for the generation of narrowband FM signals due to the assumption

$$\frac{k_0}{C_0} m(t) \ll 1.$$

The instability of the carrier frequency can be alleviated using a feedback mechanism as shown in Figure-1A.

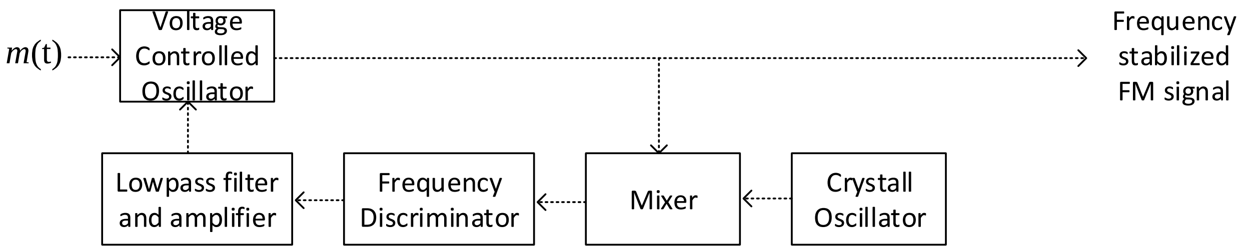


Figure-1C Frequency stabilized FM Signal Generation

On Figure-1C frequency discriminator is a device whose output voltage has an instantaneous amplitude that is proportional to the instantaneous frequency of the FM signal applied to its input.

The DC voltage at the output of the lowpass filter is applied to VCO of the FM transmitter in such a way as to modify the frequency of the oscillator in a direction that tends to restore the carrier frequency to its correct value.

WBFM signal generation using Direct Method

Since Direct Method is suitably for Narrowband FM signal generation, a WBFM signal can be generated from NBFM signal as shown in Figure-1D.

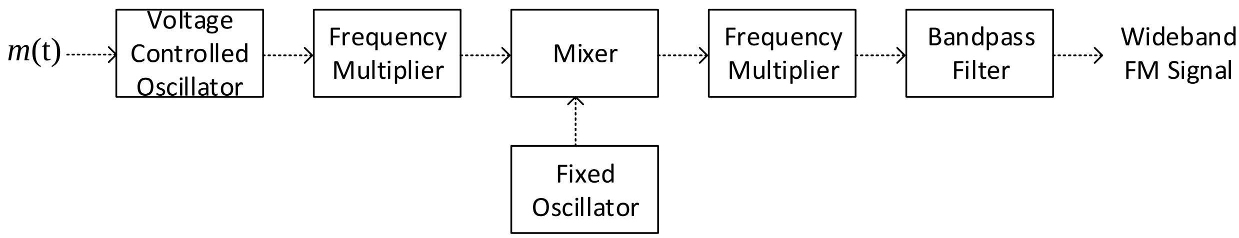


Figure-1D WBFM Signal Generation Using VCO

This method of FM signal generation suffers from carrier frequency drift.

INDIRECT METHOD for FM Signal Generation (ARMSTRONG MODULATOR, used for WBFM Signal Generation)

In the indirect method, on the other hand, the message signal is first used to produce a narrow-band FM, which is followed by frequency multiplication to increase the frequency deviation to the desired level.

In this second method, the carrier-frequency stability problem is alleviated by using a highly stable oscillator (e.g., crystal oscillator) in the narrowband

FM generation; this modulation scheme is called the Armstrong wide-band frequency modulator, in recognition of its inventor.

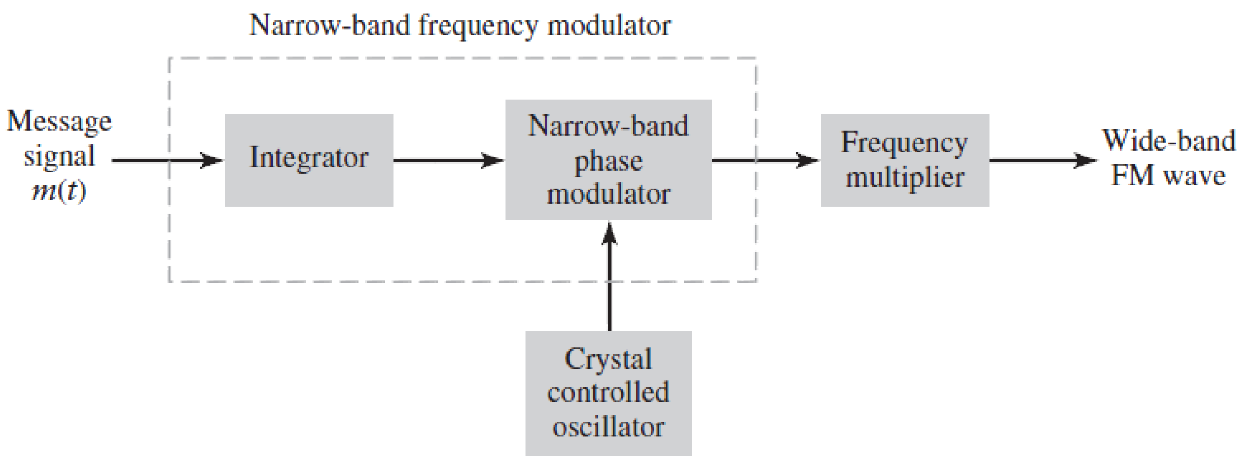


Figure-2 Block diagram of the indirect method of generating a wide-band FM wave.

The frequency multiplier block in Figure-2 is detailed in Figure-3.

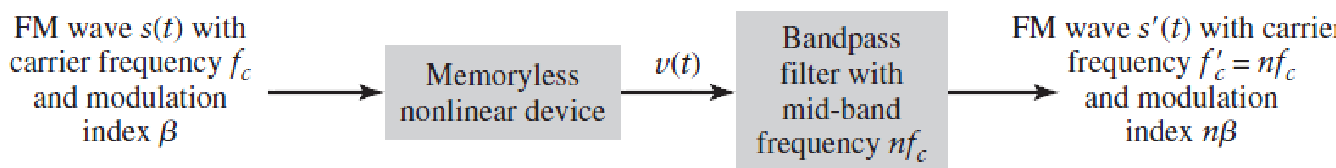


Figure-3 Block diagram of frequency multiplier.

For the non-linear device we have the input-output relation

$$v(t) = a_1 s(t) + a_1 s^2(t) + \cdots + a_n s^n(t)$$

For

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right) = A_c \cos\theta_i$$

the output of the non-linear device can be written as

$$v(t) = a_1 A_c \cos\theta_i + a_2 A_c^2 \cos^2\theta_i + a_3 A_c^3 \cos^3\theta_i + \cdots a_n A_c^n \cos^{\mathrm{n}}\theta_i$$

where employing the properties

$$\cos^2(x) = \frac{1}{2}(\cos(2x) + 1)$$
$$\cos^3(x) = \frac{1}{4}(\cos(3x) + 3\cos(x))$$
$$\cos^4(x) = \frac{1}{8}(\cos(4x) + 4\cos(2x) + 3\cos(x))$$
$$\cos^5(x) = \frac{1}{16}(\cos(5x) + 5\cos(3x) + 10\cos(x))$$

we obtain

$$v(t) = a_1 A_c \cos\theta_i + \frac{a_2 A_c^2}{2} + a_2 A_c^2 \cos 2\theta_i + \left(\frac{3a_3 A_c^3}{4}\right)\cos\theta_i + \left(\frac{3a_3 A_c^3}{4}\right)\cos 3\theta_i + \cdots$$

which can be written as

$$v(t) = \sum_{k=1}^{n} A_k \cos(k\theta_i) \rightarrow v(t) = \sum_{k=1}^{n} A_k \cos\left(2\pi k f_c t + 2\pi k k_f \int_0^t m(t)dt\right). \quad (1)$$

When (1) passes through a band-pass filter with mid-band frequency nf_c, we obtain the WBFM signal at the output of the BP filter, and the WBFM signal generated is

$$s'(t) = A_c' \cos\left[2\pi n f_c t + 2\pi n k_f \int_0^t m(t)dt\right]$$

Whose instantaneous frequency is given as

$$f_i'(t) = nf_c + nk_f m(t).$$

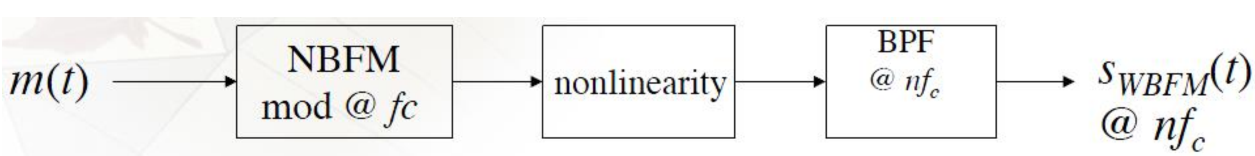


Figure-4 Generation of WBFM signal from NBFM signal (Armstrong Method)

In Figure-5 another diagram for the illustration for the generation of WBFM signal is provided.

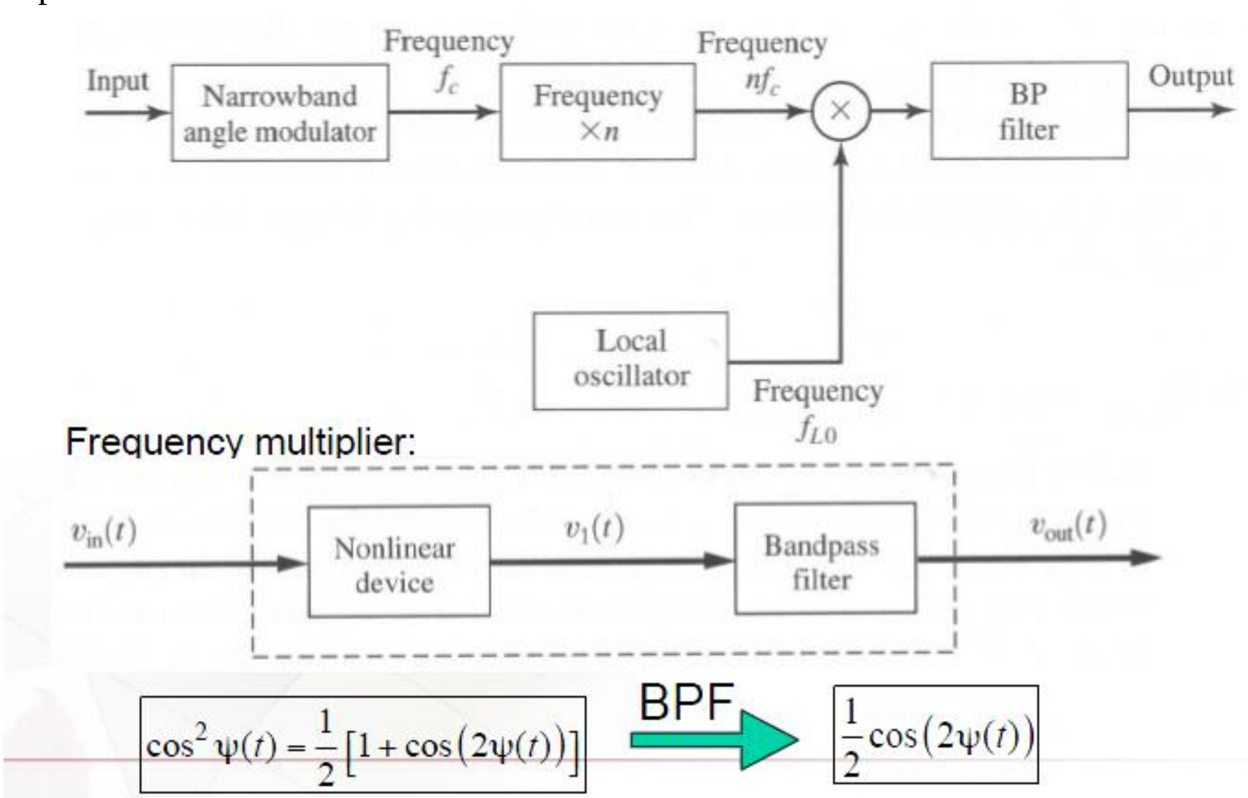


Figure-5 Generation of WBFM signal from NBFM signal (Armstrong Method)

Example: The generation of the WBFM signal for $f_c = 91.2MHz$ is illustrated in Figure-6.

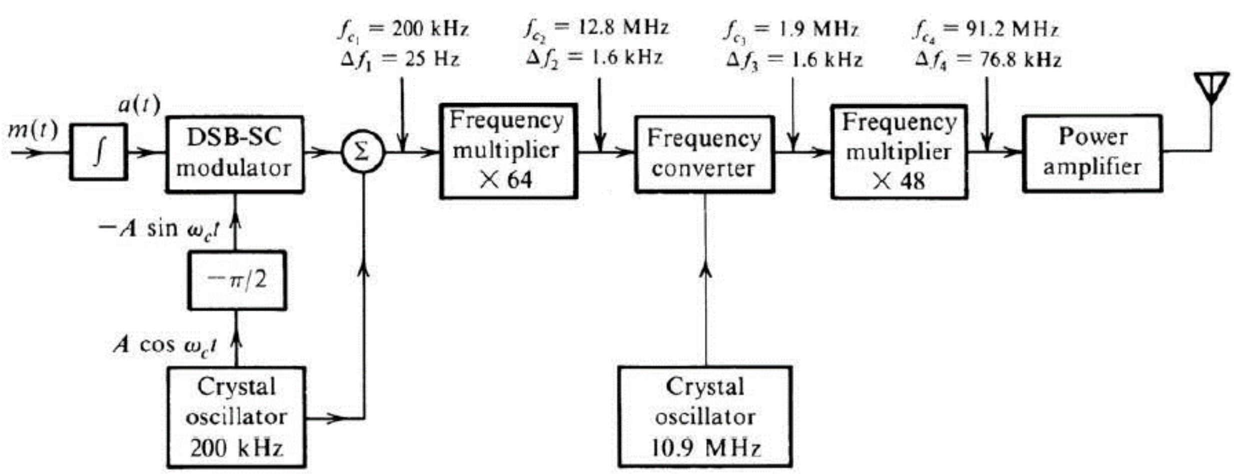


Figure-6 WBFM Signal Generation

Example: