ECE 373 Fundamentals of Communication

Lecture-15

Phase Modulation

For the modulating signal, i.e., message signal, $m(t)$, the phase modulated (PM) is obtained as

$$s(t) = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$$

Frequency Modulation

For the modulating signal, i.e., message signal, $m(t)$, the frequency modulated (FM) is obtained as

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right)$$

Narrowband and Wideband FM

For message signal $m(t)$, narrowband FM is generated as

$$s(t) = A_c \cos(2\pi f_c t) - A_c \theta(t) \sin(2\pi f_c t)$$

$$\theta(t) = 2\pi k_f \int_0^t m(t)dt$$

For narrowband modulated signals we have

$$\beta = \frac{\Delta f}{W_m} \ll 1$$

and for wideband modulated signals we have

$$\beta = \frac{\Delta f}{W_m} > 1.$$

For the message signal $m(t)$ having bandwidth W_m, the bandwidth of the narrowband modulated signal $s_{NBFM}(t)$ equals to $2W_m$.

Case Study:

The bandwidth of FM modulated signal for sinusoidal message signal

For the message signal

$$m(t) = A\cos(2\pi f_m t).$$

FM modulated signal is calculated using the formula

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right)\#$$

as

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \quad (1)$$

where β_F is the modulation index defined as

$$\beta = \frac{\Delta f}{W_m} \rightarrow \beta = \frac{Ak_f}{f_m}.$$

The Fourier transform of (1) can be calculated as

$$S(f) = \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\,[\delta(f - [f_c + nf_m]) + \delta(f + [f_c + nf_m])]. \quad (2)$$

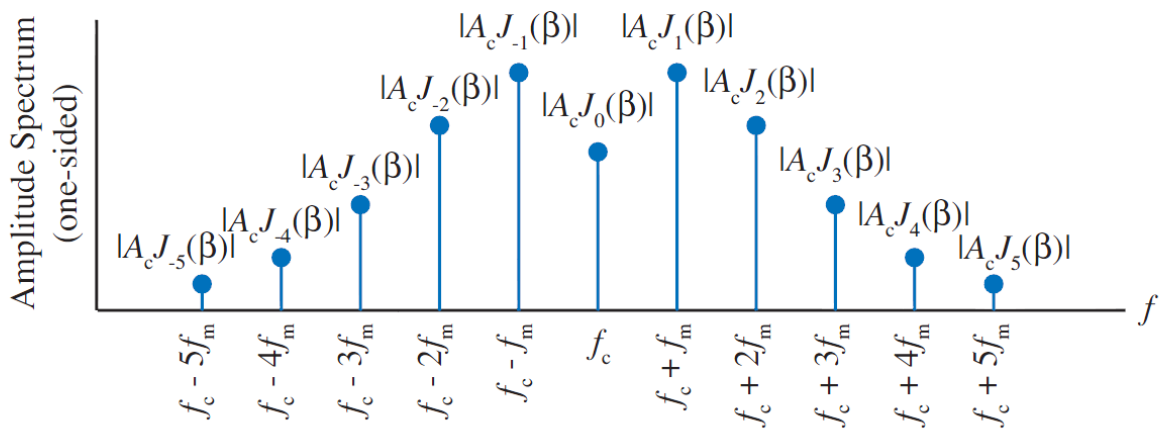
The graph of $S(f)$ consists of impulses at locations

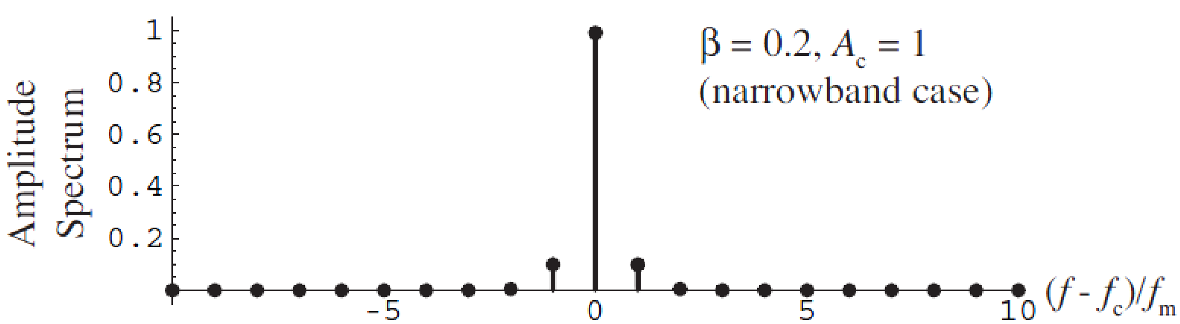
$$f = f_c + nf_m$$

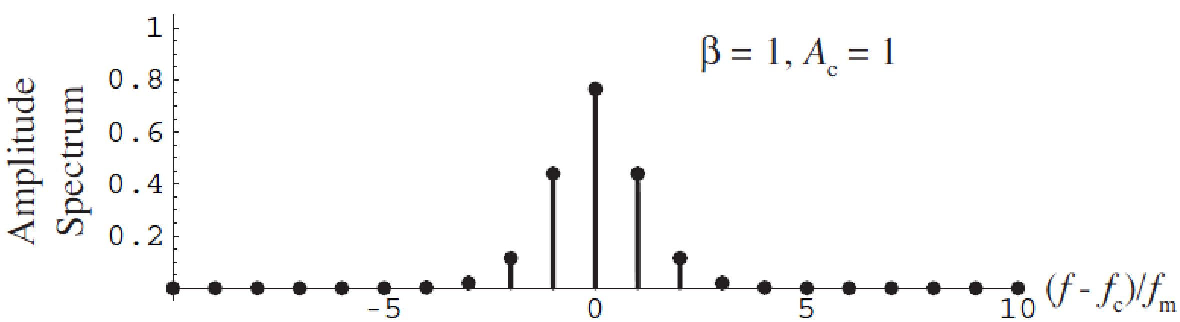
The spectrum expression

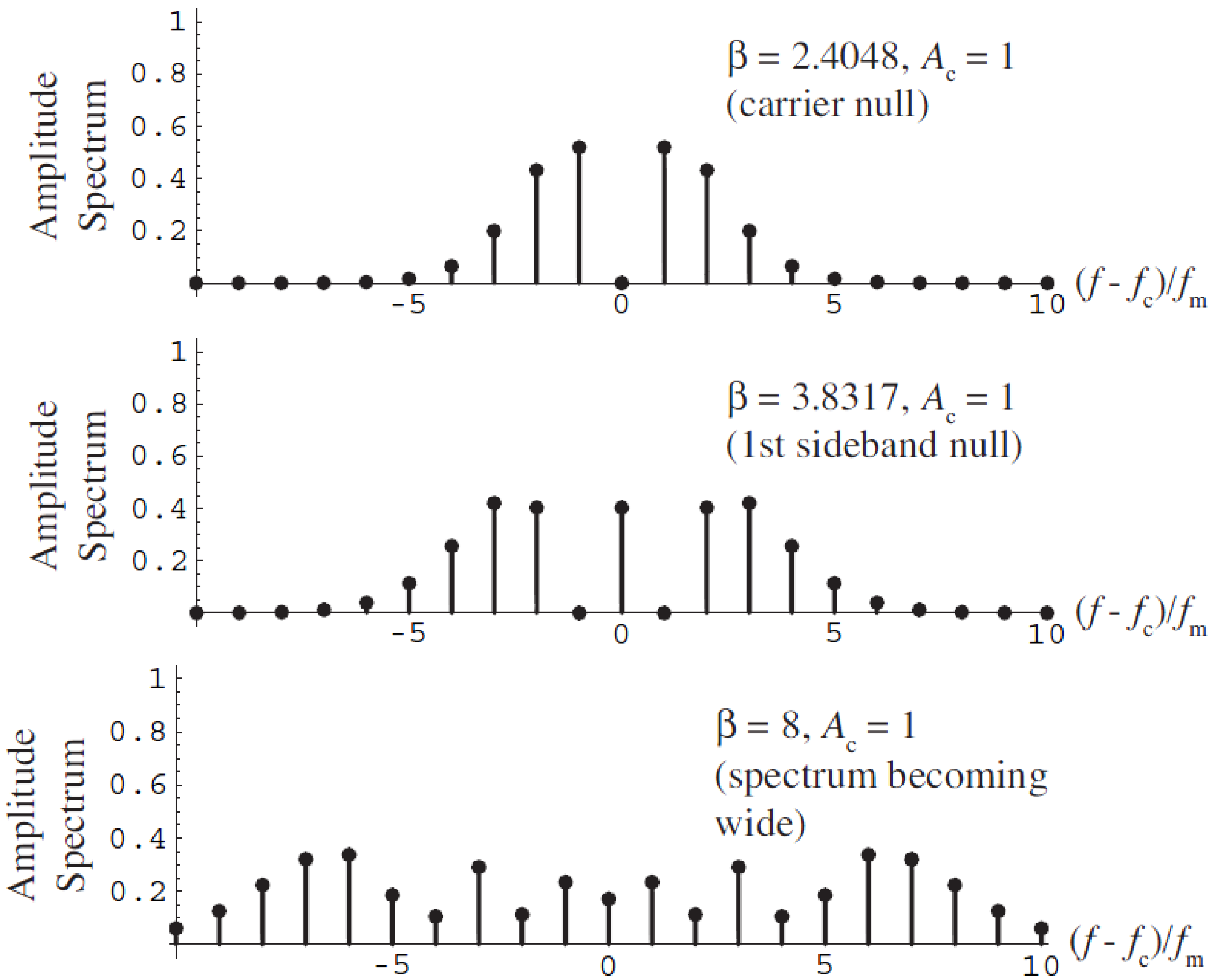
$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\,[\delta(f - [f_c + nf_m]) + \delta(f + [f_c + nf_m])]$$

Can be plotted as in Figure-1 to 3.



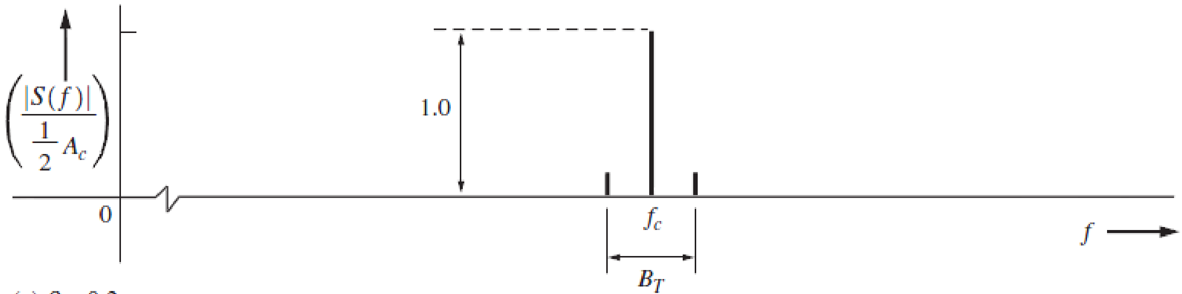




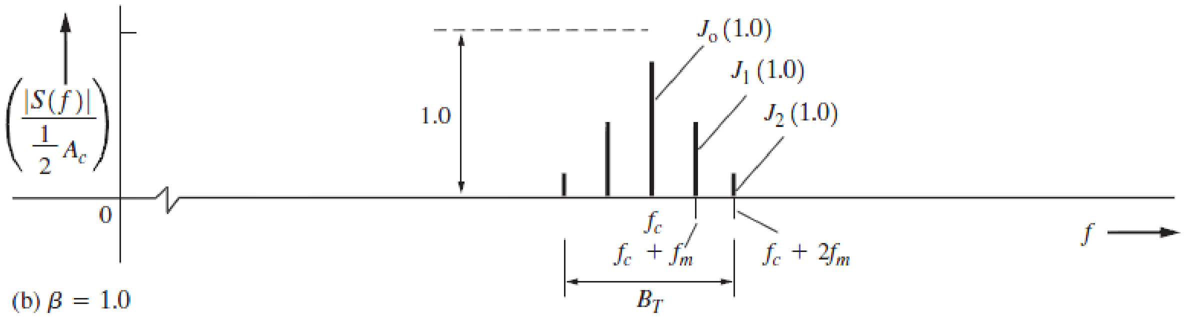


The amplitude spectrum relative to f_c as β increases

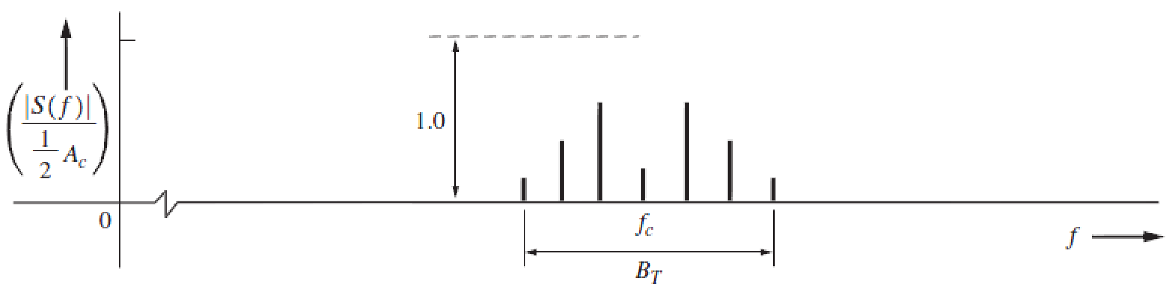
Figure-1



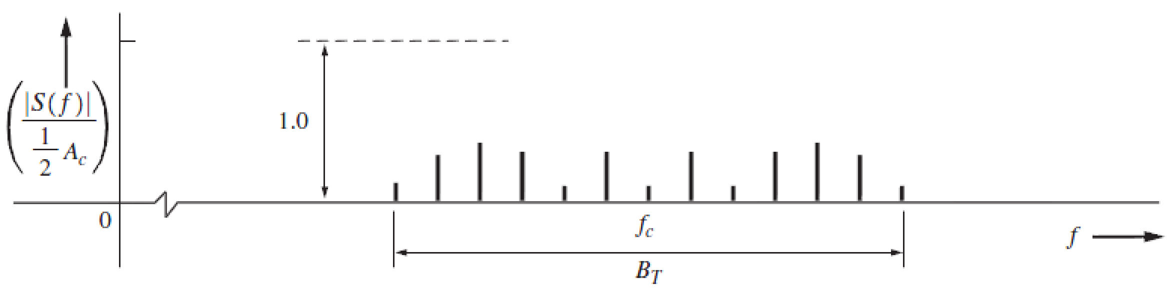
(a) $\beta = 0.2$



(b) $\beta = 1.0$



(c) $\beta = 2.0$



(d) $\beta = 5.0$

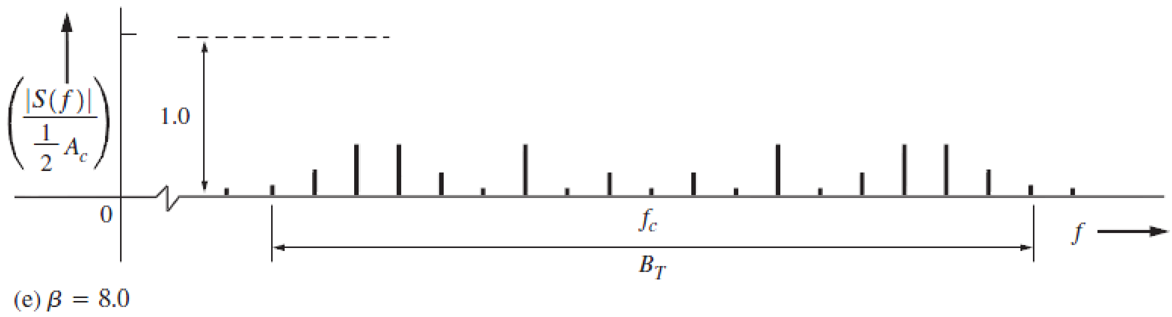


Figure-2

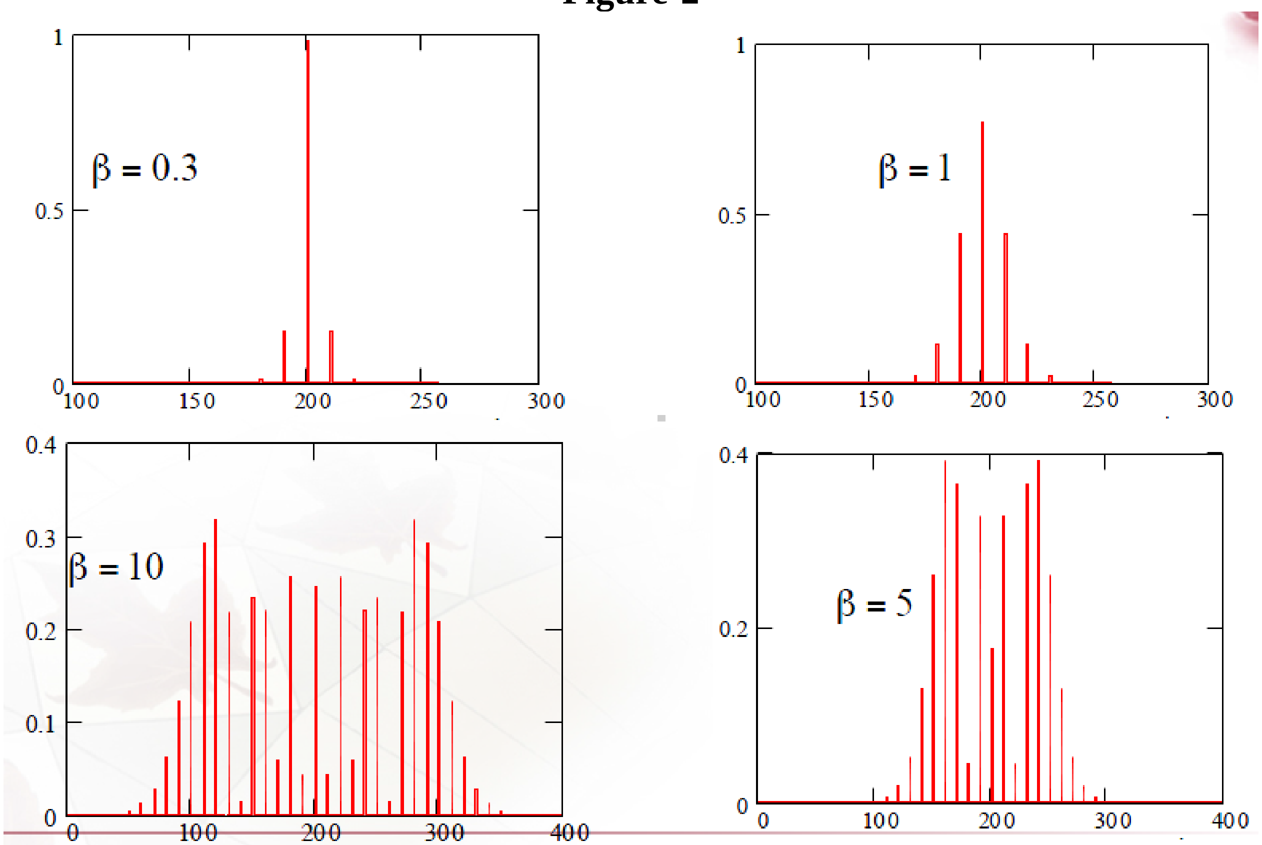


Figure-3 Carrier frequency is $f_c = 200Hz \quad f_m = 12Hz$

The graph of Bessel function $J_n(\beta)$ is depicted in Figure-4.

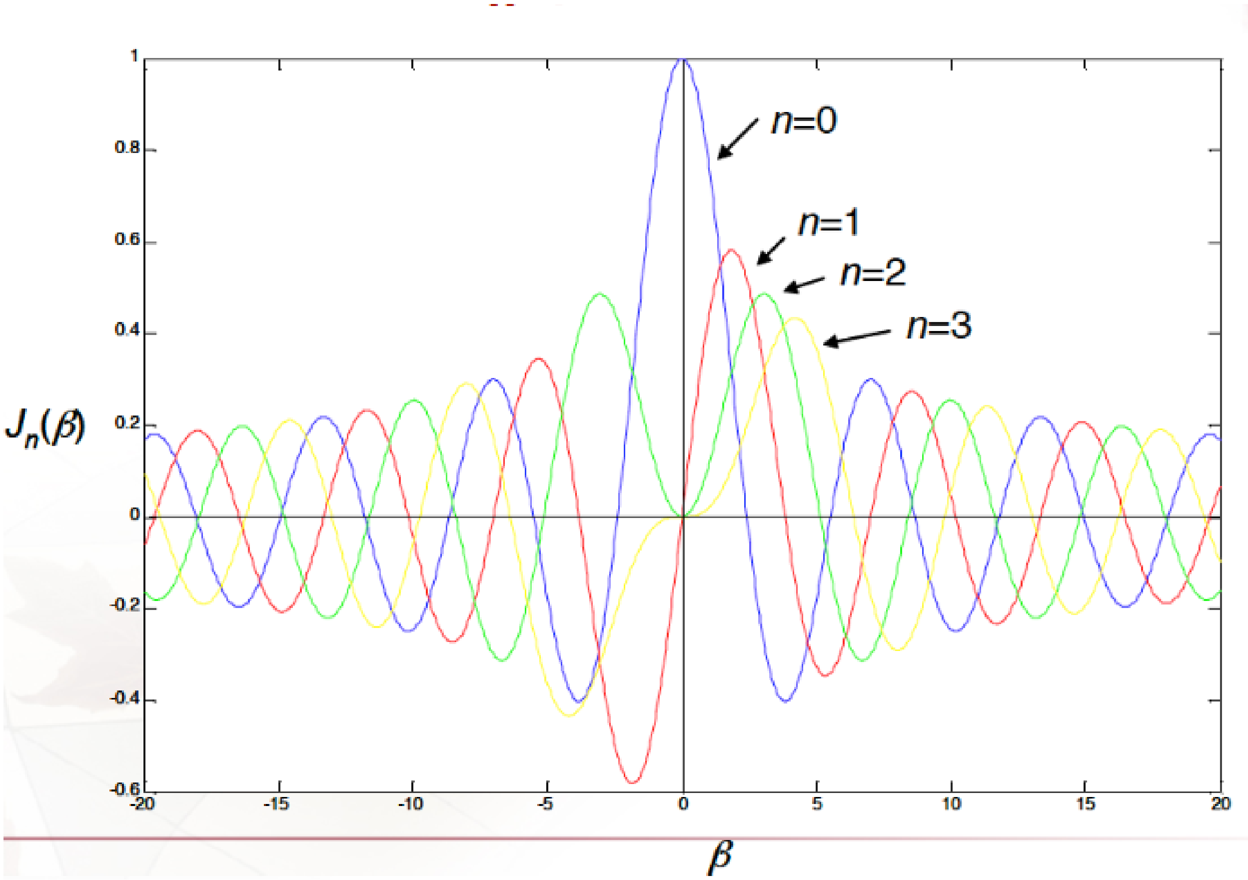


Figure-4 The graph of Bessel function $J_n(\beta)$

Properties of the Bessel function $J_n(\beta)$

1) If n is an integer :

$J_n(\beta) = J_{-n}(\beta)$ for even n

and

$J_n(\beta) = -J_{-n}(\beta)$ for odd n

2) when $\beta << 1$

$J_0(\beta) \approx 1$

$J_1(\beta) \approx \beta/2$

and

$J_n(\beta) \approx 0,\ n > 1$

4) $Im\{J_n(\beta)\}=0$

3) $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

To determine the bandwidth of the modulated signal, we can limit the summation limits of the expression

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - [f_c + nf_m]) + \delta(f + [f_c + nf_m])\right]$$

as in

$$S_N(f) = \frac{A_c}{2} \sum_{n=-N}^{N} J_n(\beta) \left[\delta(f - [f_c + nf_m]) + \delta(f + [f_c + nf_m])\right]$$

where N value is determined considering modulation index value β as in

$$J_0^2(\beta) + 2\sum_{n=1}^{N} J_n^2(\beta) \geq 0.99.$$

And the bandwidth of the modulated signal is determined as

$$2Nf_m.$$

Example: The signal $m(t) = 10cos(2\pi \times 400t)$ is to be transmitted using FM techniques. Find the practical bandwidth for $k_f = 200\ Hz/V$.

Solution: The modulation index is calculated as

$$\beta = \frac{k_f A_m}{f_m} \rightarrow \beta = \frac{10 \times 200}{400} \rightarrow \beta = 5.$$

The equation

$$J_0^2(\beta) + 2\sum_{n=1}^{N} J_n^2(\beta) \geq 0.99$$

can be solved using Table 1 for N considering the calculated $\beta = 5$ as

$$N = 6$$

and the bandwidth of the modulated signal is calculated as

$$W_s = 2Nf_m \rightarrow W_s = 12 \times 400 \rightarrow W_s = 4800Hz.$$

Table 1 Values of $J_n(\beta)$

n	β=0.1	β=0.2	β=0.5	β=1	β=2	β=3	β=5	β=10
0	0.997	0.99	0.938	0.765	0.224	-0.2601	-0.178	-0.246
1	0.05	0.1	0.242	0.44	0.577	0.3391	-0.323	0.043
2	0.001	0.005	0.031	0.115	0.353	0.4861	0.047	0.255
3	$2\times10^{-5}\approx0$	1.6×10^{-4}	0.0026	0.02	0.129	0.3091	0.365	0.058
4				0.002	0.034	0.1320	0.391	-0.220
5					0.007	0.0430	0.261	-0.234
6					0.001	0.0114	0.131	-0.014
7						0.0025	0.053	0.217
8							0.018	0.318
9							0.006	0.292
10							0.001	0.207
11								0.123
12								0.063
13								0.029

Radio Broadcast

Here are some of the major applications for AM and FM

Application	Type of Modulation
AM broadcast radio	AM
FM broadcast radio	FM
FM stereo multiplex sound	DSB (AM) and FM
TV sound	FM
TV picture (video)	AM, VSB
TV color signals	Quadrature DSB (AM)
Cellular telephone	FM, FSK, PSK
Cordless telephone	FM, PSK
Fax machine	FM, QAM (AM plus PSK)
Aircraft radio	AM
Marine radio	FM and SSB (AM)
Mobile and handheld radio	FM
Citizens band radio	AM and SSB (AM)
Amateur radio	FM and SSB (AM)
Computer modems	FSK, PSK, QAM (AM plus PSK)
Garage door opener	OOK
TV remote control	OOK
VCR	FM
Family Radio service	FM
Bluetooth radio	FSK

Figure 5

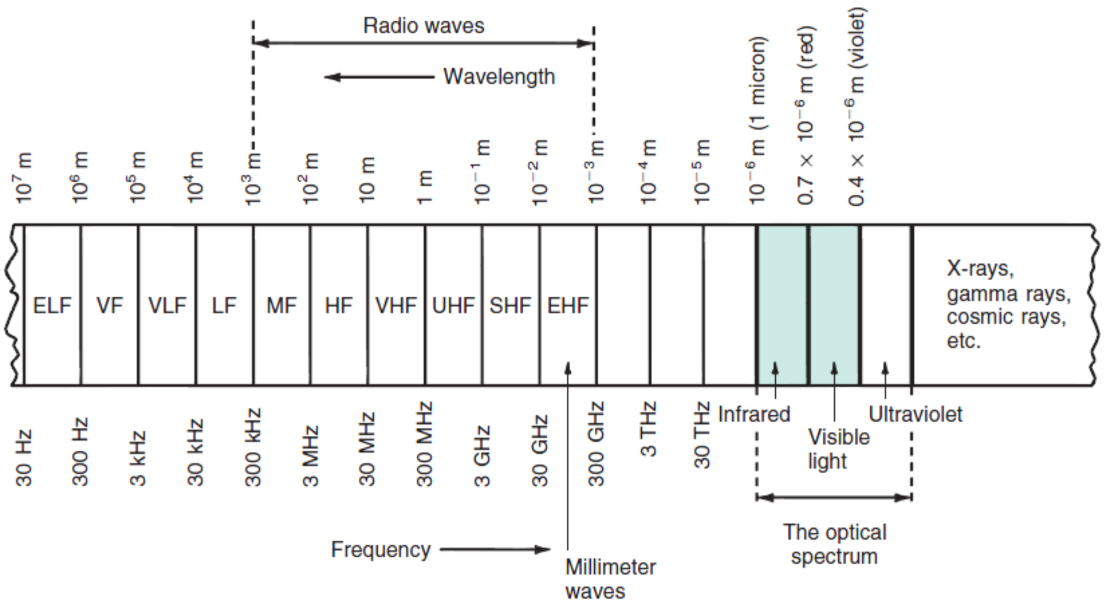


Figure 6 The electromagnetic spectrum.

Name	Frequency	Wavelength
Extremely low frequencies (ELFs)	30–300 Hz	$10^7 - 10^6$ m
Voice frequencies (VFs)	300–3000 Hz	$10^6 - 10^5$ m
Very low frequencies (VLFs)	3–30 kHz	$10^5 - 10^4$ m
Low frequencies (LFs)	30–300 kHz	$10^4 - 10^3$ m
Medium frequencies (MFs)	300 kHz–3 MHz	$10^3 - 10^2$ m
High frequencies (HFs)	3–30 MHz	$10^2 - 10^1$ m
Very high frequencies (VHFs)	30–300 MHz	$10^1 - 1$ m
Ultra high frequencies (UHFs)	300 MHz–3 GHz	$1 - 10^{-1}$ m
Super high frequencies (SHFs)	3–30 GHz	$10^{-1} - 10^{-2}$ m
Extremely high frequencies (EHFs)	30–300 GHz	$10^{-2} - 10^{-3}$ m
Infrared	—	0.7–10 μm
The visible spectrum (light)	—	0.4–0.8 μm

Units of Measure and Abbreviations:

kHz = 1000 Hz

MHz = 1000 kHz = 1×10^6 = 1,000,000 Hz

GHz = 1000 MHz = 1×10^6 = 1,000,000 kHz

= 1×10^9 = 1,000,000,000 Hz

m = meter

$\mu m = \text{micrometer} = \frac{1}{1{,}000{,}000} m = 1 \times 10^{-6} m$

Figure 6 The electromagnetic spectrum used in electronic communication.

AM Broadcasting

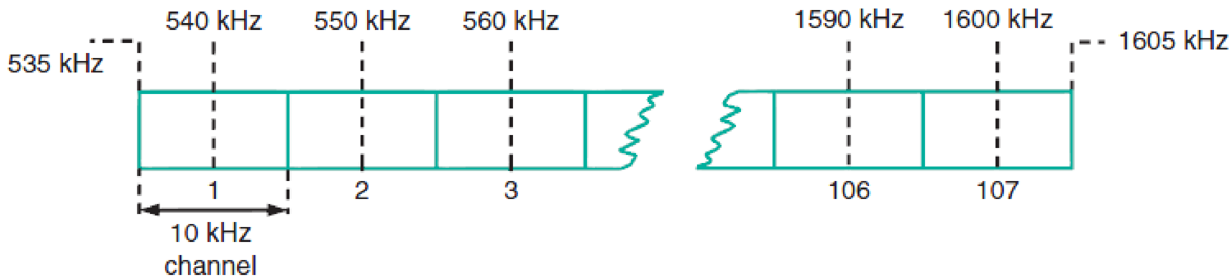


Figure 7 Frequency spectrum of AM broadcast band.

An AM broadcast station has a total bandwidth of 10 kHz.

In addition, AM broadcast stations are spaced every 10 kHz across the spectrum from 540 to 1600 kHz.

This is illustrated in Figure 7. The sidebands from the first AM broadcast frequency extend down to 535 kHz and up to 545 kHz, forming a 10-kHz channel for the signal.

The highest channel frequency is 1600 kHz, with sidebands extending from 1595 up to 1605 kHz.

There are a total of 107 10-kHz-wide channels for AM radio stations.

FM Broadcasting

For AM: 10kHz bandwidth from 540-1600 kHz for 107 possible bands.

For FM: 200 kHz bandwidth from 88.1-108.1 MHz from 100 possible bands.

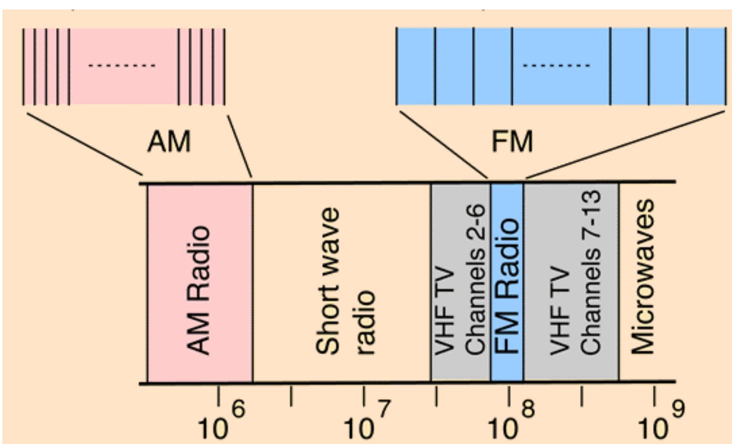


Figure 8

The FM radio broadcast band is allocated the frequency spectrum from 88 to 108 MHz.

There are 100 channels spaced 200 kHz apart. The first channel center frequency is 88.1 MHz; the last, or 100th, channel center frequency is 107.9 MHz.

The channel center frequencies are centered as

$$88.1, \quad 88.3, \quad 88.5, \quad 88.7, \cdots\cdots, 107.7, \quad 107.9.$$

Each 200-kHz channel has a 150-kHz modulation bandwidth with 25-kHz "guard bands" on either side of it.

The FM broadcast band permits a maximum deviation of ± 75 kHz and a maximum modulating frequency of 15 kHz.

Note: FM radio was assigned the 42–50 MHz band of the spectrum in 1940. In 1945, the FCC (Federal Communications Commission | The United States) moved FM to 88–108 MHz, obsoleting all existing receivers.