ECE 373 HW #2

1) An angle modulated signal is given as

$$s(t) = 50\cos(2\pi 10^6 t + 0.001\cos(2\pi 500 t))$$

a) What is $f_i(t)$, the instantaneous frequency of $s(t)$?
b) What is the approximate bandwidth of $s(t)$?
c) Is this a narrowband or wideband angle modulated signal?
d) If this represents an FM signal, what is $m(t)$, the message signal?
e) If this represents a PM signal, what is $m(t)$?

Solution: For the message signal

$$m(t) = A\cos(2\pi f_m t).$$

The PM signal can be calculated using the formula

$$s(t) = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$$

as

$$s(t) = A_c \cos\big(2\pi f_c t + k_p A\cos(2\pi f_m t)\big).$$

and the FM signal can be calculated using the formula

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right)$$

as

$$s(t) = A_c \cos\left(2\pi f_c t + \frac{A k_f}{f_m}\sin(2\pi f_m t)\right)$$

which can also be written as

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

where β is the modulation index.

a)

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i}{dt} \rightarrow f_i(t) = \frac{1}{2\pi}\frac{d}{dt}(2\pi 10^6 t + 0.001\cos(2\pi 500 t)) \rightarrow$$

$$f_i(t) = 10^6 t - 0.5\sin(2\pi 500 t)$$

b) The modulation index is $\beta = 0.001$. The bandwidth of the modulated signal can be calculated using the Carson's rule as

$$BW = 2(\Delta f + W_m) \rightarrow BW = 2W_m(1+\beta)$$

which can be approximated for $\beta \ll 1$ as

$$BW \approx 2W_m$$

where W_m is the bandwidth of the message signal. For our example

$$W_m = 500Hz$$

then

$$BW \approx 2W_m \rightarrow BW \approx 1000Hz \rightarrow BW = 1kHz$$

c) Since $\beta = 0.001 \ll 1$, this is a Narrowband FM signal.

d) If this represents an FM signal, then the message signal should be in the form

$$m(t) = A\sin(2\pi 500t)$$

e) If this represents an FM signal, then the message signal should be in the form

$$m(t) = A\cos(2\pi 500t)$$

2) What is the envelope of the signal

$$s(t) = 3\cos(2\pi 500t) + 4\sin(2\pi 100t)$$

3) The FM signal is given as

$$s(t) = 3\cos(2\pi 500t + 0.1\cos(2\pi 400t)) + 4\sin(2\pi 500t + 0.3\cos(2\pi 800t))$$

a) What is the instantaneous frequency?
b) What is the frequency deviation?
c) What is the bandwidth?

4) Major blocks of an FM transmitter is show in Figure-1. The input $m(t)$ is an audio signal bandlimited to $100Hz - 10kHz$ frequency range, and its magnitude $|m(t)|_{max} = 1$.

Modulation index of the narrowband FM generator is adjusted to a maximum of $\beta = 0.3$, and the carrier frequency is $f_{c1} = 100kHz$.

a) Sketch the details of the box labeled "Narrowband FM Generator". What is k_f, frequency deviation Δf and bandwidth at the output of the NBFM generator? Write an expression for the signal at the NBFM generator output.

b) Sketch the details of the box labeled "Frequency Converter" which will yield an FM signal at the output with carrier frequency $f_c 50MHz$. Calculate the bandwidth of the output signal.

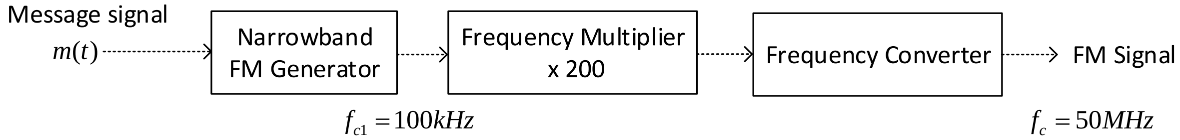

Figure-1

Solution: For message signal $m(t)$, narrowband FM is generated as

$$s(t) = A_c \cos(2\pi f_c t) - A_c \theta(t) \sin(2\pi f_c t)$$

$$\theta(t) = 2\pi k_f \int_0^t m(t)dt$$

For narrowband modulated signals we have

$$\beta = \frac{\Delta f}{W_m} \ll 1$$

a) The block diagram if the NBFM generator is shown in Figure-2.

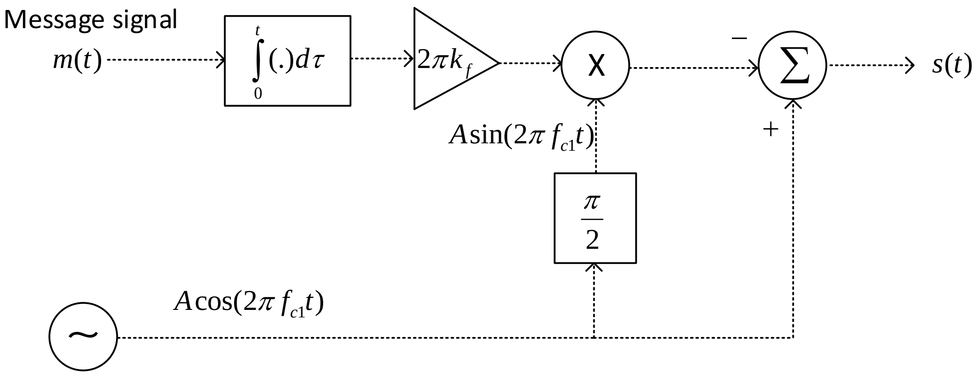

Figure-2

The signal at the output of the NBFM is

$$s(t) = A_c \cos(2\pi f_{c1} t) - A_c 2\pi k_f \int_0^t m(t)dt \sin(2\pi f_{c1} t)$$

The constant term k_f can be found as follows

$$\beta = \frac{\Delta f}{W_m} \to 0.3 = \frac{\Delta f}{100Hz} \to \Delta f = 30$$

$$\Delta f = k_f |m(t)|_{max} \to 30 = k_f \times 1 \to k_f = 30\, V/Hz$$

The bandwidth at the output of the NBFM equals to

$$BW = 2W_m(1+\beta) \approx 2W_m \to BW = 20kHz.$$

b) The frequency converter is shown in Figure-3

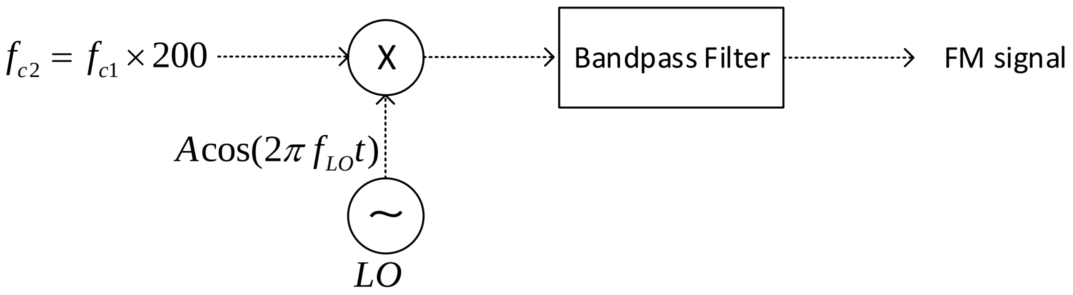

Figure-3

At the output of the frequency multiplier we have a cosine signal with frequency

$$f_{c2} = 200 \times f_{c1} \to f_{c2} = 200 \times 100kHz \to f_{c2} = 20MHz$$

It is desired to have an FM signal with frequency $f_c = 50MHz$ at the output of the modulator. Considering this, we can choose the frequency of the local oscillator as

$$f_{LO} = f_{FM} \pm f_{c2}$$

leading to

$$f_{LO} = 50 \pm 20 \to f_{LO} = 70MHz \text{ or } f_{LO} = 30MHz.$$

The frequency deviation at the input of the frequency converter is

$$\Delta f_2 = 200 \times \Delta f_1 \to \Delta f_2 = 200 \times 30Hz \to \Delta f_2 = 6kHz$$

The bandwidth of the FM signal is

$$BW = 2(\Delta f + W_m) \to BW = 2(6kHz + 10kHz) \to BW = 32kHz.$$

The bandpass filter has the center frequency $50MHz$ and bandwidth $32kHz$ so that at the output of the filter only the cosine signal with frequency $50MHz$ and bandwidth $32kHz$ is available.

4) What RMS value of

$$s(t) = A\cos(2\pi 500t + \phi)$$

Solution: RMS of the cosine signal is calculated using

$$S_{RMS} = \frac{A}{\sqrt{2}}$$

5) What is the average power of the signal

$$s(t) = A\cos(2\pi 500t + \phi)$$

Solution: The average power of cosine or sine signal with amplitude A is calculated using

$$\frac{A^2}{2}$$

which is the square of its RMS value.

6) The angle modulated signal is given as

$$s(t) = 8\cos(2\pi 10^4 t + 2\sin(2\pi 800t) + 4\cos(2\pi 1000t))$$

What is the average power of $s(t)$?

Solution: The average power of $s(t)$ can be calculated as

$$\frac{A^2}{2} \rightarrow \frac{8^2}{2} = 64W$$

7) The message signal $m(t)$ with bandwidth $500Hz$ is frequency modulated with a $10MHz$ carrier signal. The maximum frequency deviation is $50kHz$. What is the bandwidth of the FM signal?

Solution: The bandwidth can be calculated using the Carson's rule as

$$BW = 2(\Delta f + W_m) \rightarrow BW = 2 \rightarrow BW = 101kHz$$

8) A modulated signal is given as

$$s(t) = 5[\cos(2\pi(5 \times 10^5)t) - \sin(2\pi 500t)\sin(2\pi(5 \times 10^5)t)]$$

Which modulation method is used to get $s(t)$?

DSB-SC, AM, SSB, VSB, Narrowband FM, WB-FM

Solution: It is Narrowband FM signal.

9) It is claimed that the signal

$$s(t) = 5[\cos(2\pi 500t) - \sin(2\pi 200t)\sin(2\pi 400t)]$$

is a Narrowband FM signal. Is this claim true or not?

Solution: Narrowband FM signal should be in the form

$$s(t) = A_c\cos(2\pi f_c t) - A_c' m'(t)\sin(2\pi f_c t)$$

For the given signal

$$s(t) = 5[\cos(2\pi \mathbf{500}t) - \sin(2\pi 200t)\sin(2\pi \mathbf{400}t)]$$

we do not have the same frequencies for cosine and sine signals as indicated above. Hence, this is not a Narrowband FM signal. In fact, it is not a modulated signal of any type.