ECE 373 HW#1 Answers

(1) $G(f) = \dfrac{1}{1-jf} \rightarrow |G(f)| = \dfrac{1}{|1-jf|} \rightarrow |G(f)| = \dfrac{1}{\sqrt{1+f^2}}$

$$\angle G(f) = \angle 1 - \angle 1-jf$$

$$= 0 - \arctan(-f)$$

$$= \arctan(f)$$

(2) $g(t) = \delta(t+3) + \delta(t-1)$

$$G(f) = \int g(t)\, e^{-j2\pi ft}\, dt$$

$$G(f) = \int (\delta(t+3) + \delta(t-1))\, e^{-j2\pi ft}\, dt$$

$$= e^{-j2\pi f(-3)} + e^{-j2\pi f \cdot 1}$$

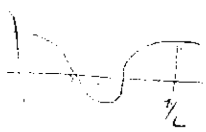
$$= e^{j6\pi f} + e^{-j2\pi f}$$

$$= e^{j2\pi f}\left[e^{j4\pi f} + e^{-j4\pi f}\right]$$

$$G(f) = e^{j2\pi f}\, 2\cos 4\pi f$$

$$|G(f)| = |e^{j2\pi f}|\,|2|\,|\cos 4\pi f|$$

Magnitude: $|G(f)| = 2\,|\cos(4\pi f)|$



Phase $\angle G(f) = \angle e^{j2\pi f} + \angle 2 + \angle \cos 4\pi f$

$$= 2\pi f + 0 + \{-\pi \quad 1/8 < f < 3/8\}$$

③ $e^{j2\pi f_0 t} \xleftrightarrow{FT} \delta(f-f_0)$

$$x(t) = e^{j2\pi t} + e^{-j4\pi t}$$

$$X(f) = \delta(f-1) + \delta(f+2)$$

④ $X(f) = \delta(f-4) + \delta(f+4) + \delta(f+8)$

$$x(t) = \int X(f)\, e^{j2\pi f t}\, dt$$

$$= e^{j2\pi 4t} + e^{j2\pi(-4)t} + e^{j2\pi(-8)t}$$

$$= 2\cos(8\pi f t) + e^{-j16\pi t}$$

⑤ $g(t) \xleftrightarrow{FT} G(f) \qquad g(at) \xleftrightarrow{FT} \frac{1}{|a|} G\left(\frac{f}{a}\right)$

$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt$$

$$g(at) \xrightarrow{FT} G_1(f) = \int_{-\infty}^{\infty} g(at)\, e^{-j2\pi f t}\, dt$$

for $a>0 \quad u=at \quad G_1(f) = \int_{-\infty}^{\infty} g(u)\, e^{-j2\pi f \frac{u}{a}} \frac{1}{a} du$ Ⓐ

$$= \frac{1}{a} \int_{-\infty}^{\infty} g(u)\, e^{-j2\pi f \frac{u}{a}} du$$

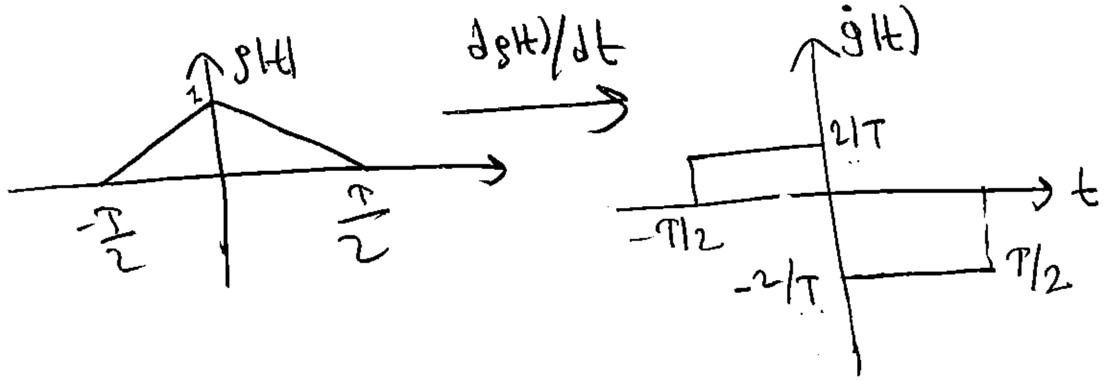
for $a<0 \quad u=at \quad G_1(f) = \int_{+\infty}^{-\infty} g(u)\, e^{-j2\pi f t \frac{u}{a}} \frac{1}{a} du$ Ⓑ

$$= -\frac{1}{a} \int_{-\infty}^{\infty} g(u)\, e^{-j2\pi f t \frac{u}{a}} du$$

from A & B we conclude that

$$g(at) \xleftrightarrow{FT} \frac{1}{|a|} G\left(\frac{f}{a}\right)$$

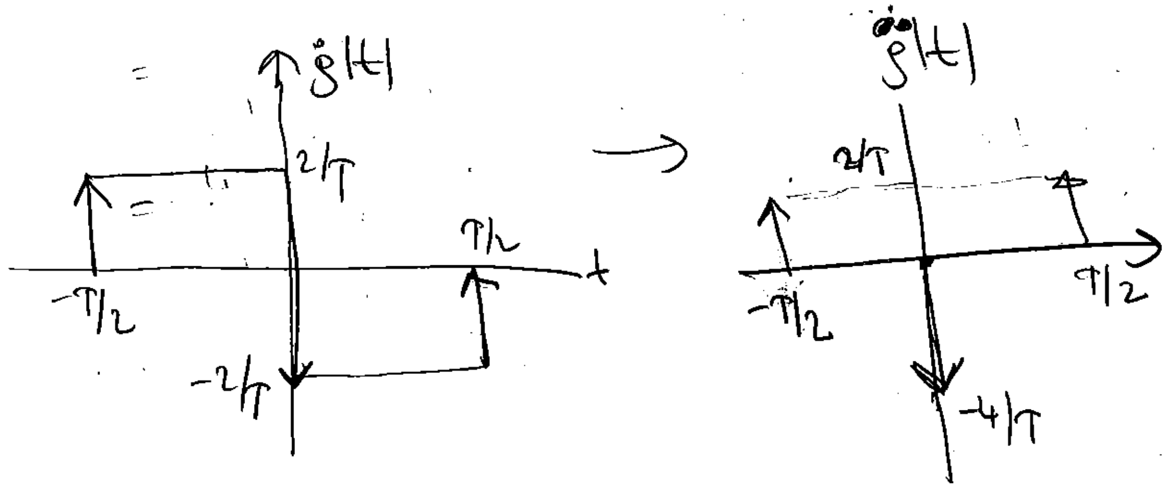
(6)



$$\dot g(t) = -\frac{2}{T}\left[u(t)-u\left(t-\frac{T}{2}\right)\right] + \frac{2}{T}\left[u(-t)-u\left(-t-\frac{T}{2}\right)\right]$$

(7) The derivative of $\dot g(t)$ can be calculated as

$$\ddot g(t) = -\frac{2}{T}\left[\delta(t)-\delta\left(t-\frac{T}{2}\right)\right] + \frac{2}{T}\left[-\delta(-t)+\delta\left(-t-\frac{T}{2}\right)\right]$$



$$\ddot g(t) = +\frac{2}{T}\delta\left(t-\frac{T}{2}\right) + \frac{2}{T}\delta\left(-t-\frac{T}{2}\right) - \frac{4}{T}\delta(t)$$

Note that $\delta(-t)=\delta(t)$

(8)

$$\frac{d^n g(t)}{dt^n} \Leftrightarrow (j2\pi f)^n G(f)$$

$$\ddot{g}(t) = \frac{2}{T}\delta\left(t-\frac{T}{2}\right) + \frac{2}{T}\delta\left(-t-\frac{T}{2}\right) - \frac{4}{T}\delta(t)$$

$$\downarrow$$

$$(j2\pi f)^2 G(f) = \frac{2}{T}e^{-j2\pi f\cdot\frac{T}{2}} + \frac{2}{T}e^{j2\pi f\frac{T}{2}} - \frac{4}{T}e^{0}$$

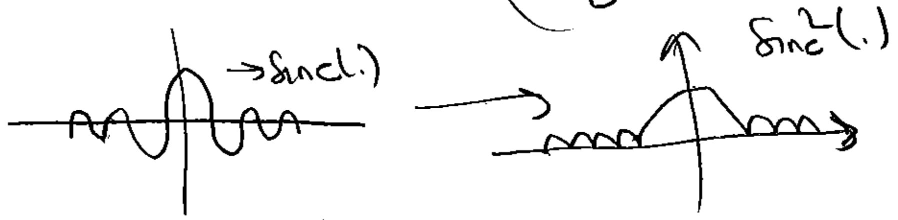
$$G(f) = \frac{1}{(j2\pi f)^2}\left[\frac{2}{T}\left[e^{j\pi fT} + e^{-j\pi fT}\right] - \frac{4}{T}\right]$$

$$G(f) = \frac{1}{(j2\pi f)^2}\left[\frac{4}{T}\cos\pi fT - \frac{4}{T}\right]$$

$$G(f) = \frac{4}{T}\frac{1}{(j2\pi f)^2}\left[\underbrace{\cos\pi fT}_{1-2\sin^2\frac{\pi fT}{2}} - 1\right] \qquad (\cos 2x = 1 - 2\sin^2 x)$$

$$G(f) = \frac{\cancel{4}}{T}\frac{1}{(-1)\cancel{4}\pi^2 f^2}(-2)\sin^2\left(\frac{\pi fT}{2}\right)$$

$$G(f) = \frac{T}{2}\,\frac{\left(\sin\frac{\pi fT}{2}\right)^2}{\left(\frac{\pi fT}{2}\right)^2} \to G(f) = \frac{T}{2}\,\text{sinc}^2\left(f\frac{T}{2}\right)$$



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$$g(t) = \underbrace{\cos(2\pi t)}_{x(t)} \, \underbrace{\cos(8\pi t)}_{y(t)}$$

$$X(f) = \frac{1}{2}\left[\delta(f-1) + \delta(f+1)\right]$$

$$Y(f) = \frac{1}{2}\left[\delta(f-4) + \delta(f+4)\right]$$

Notes: $\cos 2\pi f_0 t \xleftrightarrow{FT} \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$

$$G(f) = X(f) \otimes Y(f)$$

$$= \frac{1}{4}\left\{ \left[\delta(f-1) + \delta(f+1)\right] \otimes \left[\delta(f-4) + \delta(f+4)\right] \right\}$$

$$= \frac{1}{4}\left\{ \delta(f-1) \otimes \delta(f-4) + \delta(f-1) \otimes \delta(f+4) + \delta(f+1) \otimes \delta(f-4) + \delta(f+1) \otimes \delta(f+4) \right\}$$

$$= \frac{1}{4}\left[\delta(f-5) + \delta(f-3) + \delta(f-3) + \delta(f+5)\right]$$

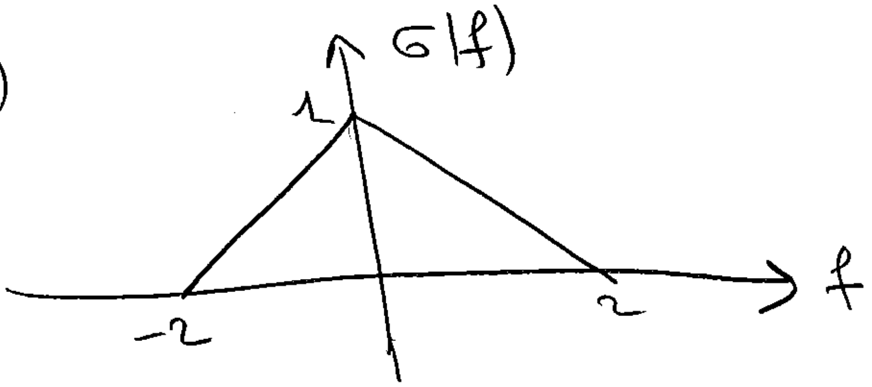
$$= \frac{1}{4}\left[\delta(f-5) + \delta(f+5) + 2\delta(f-3)\right]$$

Notes:

$$X(f) \otimes \delta(f-f_0) = X(f-f_0)$$

$$\delta(f-1) \otimes \delta(f-4) = \delta(f-4-1) \rightarrow \delta(f-5)$$

(10)



$$s(t) = g(t)\cos 16\pi t$$

$$S(f) = G(f) \otimes FT\{\cos 16\pi t\}$$

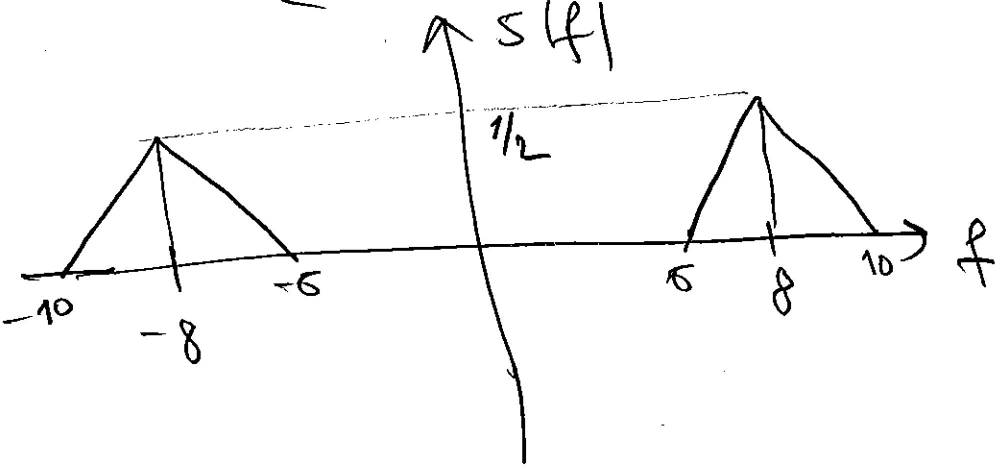
$$\cos 16\pi t \xrightarrow{FT} \frac{1}{2}\left[\delta(f-8) + \delta(f+8)\right]$$

Notes $\cos 2\pi f_0 t \xrightarrow{FT} \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$

Then we have

$$S(f) = G(f) \otimes \frac{1}{2}\left[\delta(f-8) + \delta(f+8)\right]$$

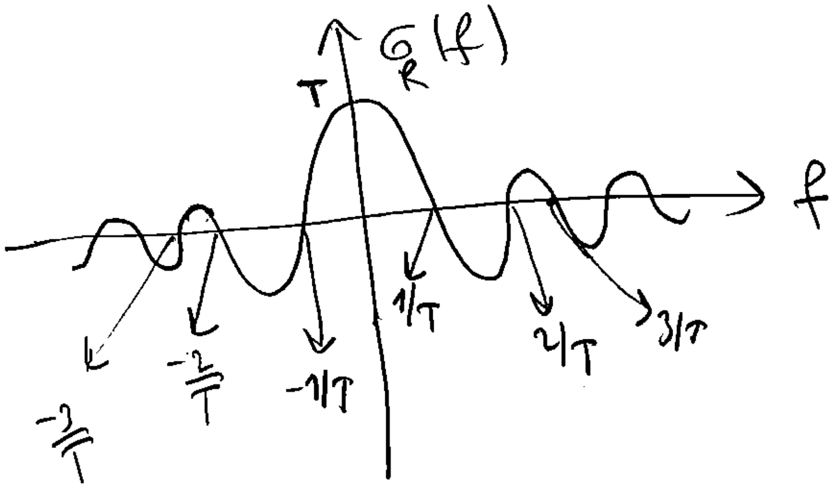
$$S(f) = \frac{1}{2}\left[G(f-8) + G(f+8)\right]$$



(11)

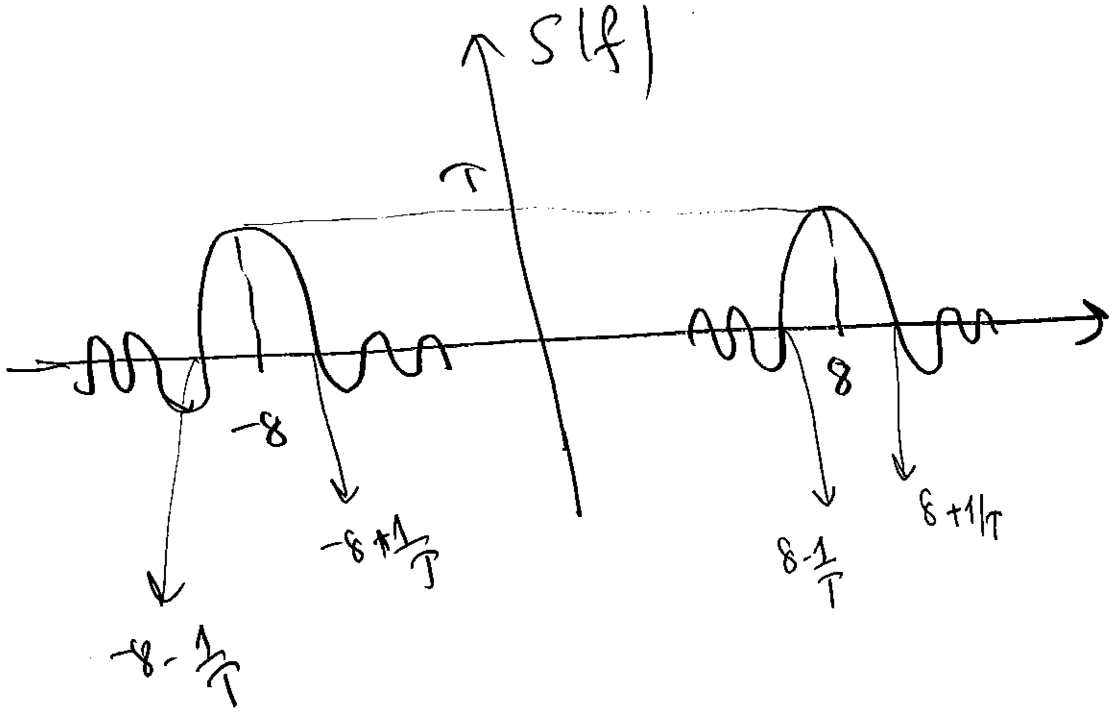
$$G_R(f) = T\,\mathrm{sinc}(fT)$$

$$= T\,\frac{\sin \pi f T}{\pi f T}$$



$$s(t) = g_R(t)\cos 16\pi t$$

$$S(f) = \frac{1}{2}\left[G_R(f-8) + G_R(f+8)\right]$$



(12)

$g(t) = m(t) \sin 2\pi f_0 t$

$G(f) = M(f) * \frac{1}{2j}\left[\delta(f - f_0) - \delta(f + f_0)\right]$

$\hat{G}(f) = -j\,\mathrm{sgn}(f)\,G(f)$

$= M(f) * \left(-j\,\mathrm{sgn}(f)\right)\left(\frac{1}{2j}\right)\left[\delta(f - f_0) - \delta(f + f_0)\right]$

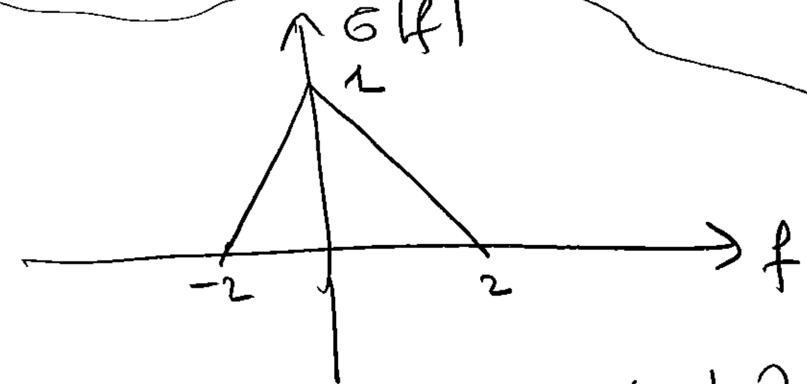
$= M(f) * -\frac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$

$\rightarrow \hat{g}(t) = -m(t) \cos 2\pi f_0 t$

Note: $\mathrm{sgn}(f)\,\delta(f - f_0) = \delta(f - f_0)$

$\mathrm{sgn}(f)\,\delta(f + f_0) = -\delta(f + f_0)$

(13)



$s(t) = \mathrm{Re}\left\{ g(t)\, e^{j16\pi t} \right\}$

$s(t) = \mathrm{Re}\left\{ g(t)\, e^{j2\pi f_c t} \right\}$

$S(f) = \frac{1}{2}\left[G(f - f_c) + G(f + f_c) \right]$